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Multiple correlation coefficient:-

whenever we are interested in studing the joint effect of a group of variables upon a variable not included in the group, our study is that of multiple correlation coefficient

$$R^2_{1.23} = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1-r_{23}^2}$$

similarly

$$R^2_{2.13} = \frac{r_{12}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23}}{1-r_{13}^2}$$

$$R^2_{3.12} = \frac{r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23}}{1-r_{12}^2}$$

partial correlation cofficient:-

correlation between any two variables studied partially i.e. studied after eliminating the linear effected of others ~~from~~ from them is called partial correlation cofficient.

$$r_{12\cdot3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}$$

$$r_{13\cdot2} = \frac{r_{13} - r_{12}r_{32}}{\sqrt{(1-r_{12}^2)(1-r_{23}^2)}}$$

$$r_{23\cdot1} = \frac{r_{23} - r_{21}r_{31}}{\sqrt{(1-r_{21}^2)(1-r_{31}^2)}}$$

where $r_{12}, r_{13}\, r_{23}$ are the correlation coefficien between x_1 and x_2 : x_1 and x_3 : X_2 and x_3 respectivly

2Q) Gram - schmidt orthogonalization process.

Statement:- Let $\{X_1, X_2 \ldots X_r\}$ be the basis of avector space $V(R)$ defined $\{Y_1, Y_2 \ldots Y_r\}$ independent by y_k.

$$y_k = x_k - \sum_{j=1}^{k-1} \frac{\langle x_k, y_j\rangle}{\langle y_j - y_j\rangle} y_j \quad \forall\, k = 1, 2 \ldots r \text{ then}$$

$\{z_1, z_2 \ldots z_r\}$ is an orthogonal basis of $V(R)$

proof:- Given than $\{x_1, x_2 \ldots x_r\}$ be the basis of vector space $V(R)$. Let $y_1 = x_1 \rightarrow (1)$ $y_2 = x_2 + a y_1 \rightarrow (2)$

For the vector y_1, y_2 to be orthogonal we should have

$\langle y_1, y_2 \rangle = 0$

$\Rightarrow \langle y_1, x_2 + a y_1 \rangle = 0$

$\langle y_1 \cdot x_2 \rangle + a \langle y_1 \cdot y_1 \rangle = 0$

$a \langle y_1 \cdot y_1 \rangle = -\langle y_1 \cdot x_2 \rangle$

$$a = \frac{-\langle y_1 \cdot x_2 \rangle}{\langle y_1 \cdot y_1 \rangle}$$

from eqn (2)

$$y_2 = x_2 + \frac{\langle y_1, x_2 \rangle y_1}{\langle y_1 \cdot y_1 \rangle}$$

Let $y_3 = x_3 + a y_2 + b y_1$ — (3)

Inner product of $\langle y_1, y_2 \rangle = 0$ $\langle y_2, y_3 \rangle = 0$ $\langle y_1, y_3 \rangle$

Now $\langle y_1, y_3 \rangle = 0$

$\langle y_1, x_3 + a y_2 + b y_1 \rangle = 0$

$$b = \frac{-\langle y_1, x_3 \rangle}{\langle y_1, y_1 \rangle}$$

Now taking $\langle y_2, y_3 \rangle = 0$ using eq (3) similarly

we get $a = \dfrac{-\langle y_2, x_3 \rangle}{\langle y_2, y_2 \rangle}$

From a & b values sub in (3)

$$y_3 = x_3 - \frac{\langle y_2, x_3 \rangle}{\langle y_2 \, y_2 \rangle} - \frac{\langle y_1 \cdot x_3 \rangle}{\langle y_1, y_1 \rangle} y_1$$

Similarly we get

$$y_2 = x_r - \frac{\langle y_{r-1}, x_r \rangle}{\langle y_{r-1}, y_{r-1} \rangle} y_{r-1} - \frac{\langle y_{r-2}, x_r \rangle}{\langle y_{r-2}; y_{r-2} \rangle} y_{r-2} - \cdots$$

$$\cdots \frac{\langle y_1, x_r \rangle}{\langle y_1, x_r \rangle} y_1$$

Here $y_1, y_2 \ldots y_r$ are mutully orthogonal Now can convert vector $y_1, y_2, y_3 \ldots y_r$ into orthonormal vector by defining $z_r = \frac{y_r}{\|y_i\|}$ $\forall$ $r = 1, \ldots r$

where $\|y_i\|$ is the length of the vector

The vector $z_1, z_2 \ldots z_r$ are mutually orthogonal basis for the vector space $V(R)$

10) Dimension:-

Defination:-

The number of elements in a basis for a finitely generated vector space V is called the dimension of V and denoted dim V.

Examples

$V = \{a_2x^2 + a_1x + a_0 \mid a_i \in R\}$ then

$\{x^2, x, 1\}$ is a basis.

$\dim(V) = 3$

Let $V = M_2(R)$ then.

$v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$v_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ is basic and $\dim(V) = 4$

Vector space:-

Defination:-

The set of vectors $S = \{v_1, v_2, \ldots v_n\} \subseteq V$ in vector space V is called a basis for V if..

- S spans V (i.e., span (S) = V}
- S is linearly independent

$\Rightarrow$ S is called a basis for V

* Notes :

(1) $\emptyset$ is basis for $\{0\}$

(2) the standard basis for R^3.

$\{i, j, k\}$ $i = (1,0,0)$. $j = (0,1,0)$ $k = (0,0,1)$

8Q: Role of orthogonal polynomial Regression:-

In fitting the polynomial model in one variable even if condition is removed by centering we may still have high level of multiple correlation [co-linearity] which cause difficulty in ~~the~~ variable, the column of x in analysis and can be eliminated by

→ A polynomial ~~in one variable the~~ using orthogonal polynomial regession ~~to~~ fit and model.

A polynomial model in one variable the column of 'x' matrix are not orthogonal $y = \beta_0 + \beta_1 x_1 + \beta_1 x_2^2 + \beta$

$+ \beta_k x^k + \varepsilon \rightarrow (1)$

$$X = \begin{bmatrix} 1 & x_2 & x_1^2 & \cdots & x_1^k \\ 1 & x_2 & x_2^2 & \cdots & x_2^k \\ \vdots & \vdots & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^k \end{bmatrix}$$

According to the least squre method one can obtain the $\vec{\beta}$ for the model in eq^n (1) but if there is araised a Need to the add one more term $\beta_{k+1}x^{k+1}$ then we have to compute regression we can restore the $\times B_0, \vec{\beta}_1, \ldots \vec{\beta}_{k+1}$ and we need to find only $\vec{\beta}_{k+1}$

★ Application :

1) To reduce multi colinearity
2) We can add x_{k+1} variable to the eqn
3) calculation can be done easily

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partial correlation coefficient:-

In a trivate set the partial correlation co-efficient between x_1 and x_2 eliminating the effect of x_3.

Mathematically. It is denoted by $r_{12\cdot3}$ and is denoted and defined by

$$r_{12\cdot3} = \frac{cov(x_{1\cdot3}\ x_{2\cdot3})}{\sqrt{V(x_{1\cdot3})\, V(x_{2\cdot3})}} \longrightarrow (*)$$

Derivation:-

consider $cov(x_{1\cdot3}\ x_{2\cdot3}) = E\{x_{1\cdot3} - E(x_{1\cdot3})\}[x_{2\cdot3} - E(x_{2\cdot3})\}$

$= E(x_{1\cdot3}\ x_{2\cdot3})$

$= \frac{1}{N}\Sigma x_1\ x_{2\cdot3} = \frac{1}{N}\Sigma x_1(x_2 - b_{23}x_3)$

$cov(x_{1\cdot3}x_{2\cdot3}) = \frac{1}{N}\Sigma x_1x_2 - \frac{1}{N}\Sigma x_1 b_{2\cdot3}x_3$

$= \cancel{\frac{1}{N}\Sigma_1 x_1.}\ \ \cancel{E(x_{1\cdot3}\ x_{2\cdot3})}$

$= \frac{1}{N}\cancel{(x_{1\cdot3}\ x_{2\cdot3}) =} \frac{1}{N}\Sigma x_1\ x_{2\cdot3}$

$= \frac{1}{1} = \frac{1}{N}\Sigma x_1x_2 - \frac{1}{N}\Sigma x_1 b_{23}x_3$

$= \frac{1}{\cancel{N}}\cancel{\Sigma x} = r_{12}\sigma_1\sigma_2 - \sigma_{23}\, r_{13}\sigma_1\sigma_3$

$= r_{12}\sigma_1\sigma_2 - \sigma_{23}\cdot\frac{\sigma^2}{\sigma_3}\ \sigma_{13}\ \sigma_1\sigma_3$

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$$= r_{12}\sigma_1\sigma_2 - r_{23}\cdot\frac{\sigma_1}{\cancel{\sigma_3}}\,\cancel{r_{13}}\,\sigma_1\,\cancel{\sigma_3}$$

$$= \sigma_1\sigma_2(r_{12} - r_{13}r_{23}) \quad (1)$$

$$V(X_{1\cdot3}) = \frac{1}{N}\Sigma X_{1\cdot3}^2$$

$$= \frac{1}{N}\Sigma X_{1\cdot3}X_{1\cdot3}$$

$$= \frac{1}{N}\Sigma X_1 X_{1\cdot3}$$

$$= \frac{1}{N}\Sigma X_1(X_1 - b_{13}X_3)$$

$$= \frac{1}{N}\Sigma X_1^2 - \frac{1}{N}\Sigma X_1 b_{13}X_3$$

$$= \sigma_1^2 - b_{13}\cdot r_{13}\sigma_1\sigma_3$$

$$= \sigma_1^2 - r_{13}\frac{\sigma_1}{\sigma_3}(r_{13}\sigma_1\sigma_3)$$

$$= \sigma_1^2(1 - r_{13}^2) \quad (2)$$

Similarly $V(X_{2\cdot3}) = \sigma_2^2(1 - r_{23}^2)$ (3)

From (1) (2) (3) substitute in eq (*)

$$r_{12\cdot3} = \frac{\sigma_1\sigma_2(r_{12} - r_{13}r_{23})}{\sqrt{\sigma_1^2(1-r_{13}^2)\,\sigma_2^2(1-r_{23}^2)}}$$

$$r_{12\cdot3} = \frac{(r_{12} - r_{13}r_{23})}{\sqrt{\sigma_1^2(1-r_{13}^2)\,\sigma_2^2(1-r_{23})^2}}$$

$$r^2_{12\cdot3} = \frac{(r_{12} - r_{13}r_{23})}{(1-r_{13}^2)(1-r_{23}^2}$$

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Test of significance of partical correlation coefficient.

Let $r_{12.34\ldots(k+2)}$ be the sample partial correlation coefficient of order 'k' under assumption that the sample is drawn from multivariate normal population. Them for testing H_0.

H_0: The sample partial correlation coefficient is insignificant.

H_1: The sample partial correlation is significant

The test statistic is given by

$$|t| = \frac{|r_{12.34\ldots(k+2)}|\sqrt{n-k-2}}{\sqrt{1-r^2_{12.34\ldots(k+2)}}} \sim t_{(n-k-2)}\ df$$

Reject the H_0 if calalated $|t|$ is greater than critial value of 't' at α % LOS.

The sample partial correlation coefficient is significant

14 Gauss Mark off theorem :-

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Statement:- For any linear model $Y = X\beta + \varepsilon$ where $E(\varepsilon) = 0$, $D(\varepsilon) = V(\varepsilon) = E(\varepsilon'\varepsilon) = \sigma^2 I$. The Blue of an estimation parametric function $\hat{\psi} = \lambda'\beta$ =
$\psi = \lambda'\beta = \sum_{i=0}^{p} (\lambda_0\beta_0 + \lambda_1\beta_1 + \dots + \lambda_p\beta_p)$ is given by $\hat{\psi} = \lambda'\hat{\beta} = \sum_{i=0}^{p} (\lambda_0\hat{\beta}_0 + \lambda_1\hat{\beta}_1 + \dots \lambda_p\hat{\beta}_p)$ where $\hat{\beta}$ is the Blue of β, obtained by method of least square and unbaised of σ^2 is given by

$$\sigma^2 = \frac{(Y - X\hat{\beta})'(Y - X\hat{\beta})}{(n-p-1)}$$

proof:- Let $X'\beta$ be an estimation parameteric function and $\beta = (X'X)^{-1}X'Y$ is a BLUE of β obtained by principles of least square.

Let $\lambda'\beta = \psi = \lambda' E(\hat{\beta})$ $[E(\hat{\beta}) = \beta]$

$\therefore \lambda'\hat{\beta}$ is the unbaised of $\lambda'\beta$

let $U'Y$ be the other linear unbaised estimator of $\lambda'\beta$

$$E(U'Y) = \lambda'\beta$$

$$E(U'(X\beta + \varepsilon)) = \lambda'\beta$$

$$U'(X\beta + E(\varepsilon)) = \lambda'\beta$$

$$U'(X\beta) = \lambda'\beta$$

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$\lambda' = u'X$

let $u'y = \lambda'\beta + u'y - \lambda'\beta$

$V(u'y) = V(x'\beta) + V(u'y - \lambda'\beta) + 2\,cov(\lambda'\beta, (u'y - \lambda'\beta))$

$\Rightarrow V(u'y) = V(\lambda'\beta) + V(u'y - \lambda'\beta)$

$\therefore V(u'y) \geq V(\lambda'\beta)$

$\therefore \lambda'\beta$ is the BLUE

(ii) Now we have to prove that $\sigma^2 = \frac{(Y - X\hat{\beta})'(Y - X\hat{\beta})}{n-p-1}$

consider The error of the model $\varepsilon = Y - \hat{Y}$

$\varepsilon = y - X\hat{\beta}$

$\varepsilon'\varepsilon = (Y - X\hat{\beta})'(Y - X\beta) = \sum_{i=1}^{n} \varepsilon_i^2$

W.K.T $E(\varepsilon'\varepsilon) = \sigma^2 I$

$\varepsilon = y - X\hat{\beta}$

$= Y - X(X'X)^{-1}X'y$

$= (I - X(X'X)^{-1}X')y$

$= My$

$\therefore \varepsilon' = (Y - X\hat{\beta})'$

$= (Y - X(X'X)^{-1}X'y)'$

$= (I - X(X'X)^{-1}X')y'$

$= M'y'$

$\therefore My = M'y' \Rightarrow m = m'$

$\therefore M^2 = M.M = (I - X(X'X)^{-1}X')(I - X(X'X)^{-1}X')$

$= I^2 - IX(X'X)^{-1}X' - X(X'X)^{-1}X'I + X(X'X)^{-1}X'X(X'X)$

$= I - X(X'X)^{-1}X'$

$= M$

$\therefore M$ is Idempotent Matrix

~~The E(Q C~~

$\varepsilon'\varepsilon = y'M'My = y'My$ is quadratic form in y where M is a symmetric and idempoten matrix

The $E(Q.F) = \sigma^2$ term)

$= \sigma^2[tr[I - X(X'X)^{-1}X']]$

$= \sigma^2(n-(P+1))$

$E(\varepsilon'\varepsilon) = \sigma^2(n-P-1)$

$$\sigma^2 = \frac{E(\varepsilon'\varepsilon)}{n-P-1}$$

$$\sigma^2 = \frac{[(y - X\bar{\beta})'(y - X\bar{\beta})}{(n-P-1)}$$

Hence proved

15 Dummy variable (or) Indicate Variable

→ variabbles are of two types

i) Quantitative /metric variables : This variable have will defined scalar measurements eg:- height, weight marks cost of an product age ect

ii) Qualitative or categorical (or) non metric variable.

Eg: Gender, smoking cast, driver ectc are eg of qualitative data. which are not supplied as numerical data

→ In regression analysis we generally use qualitative variable but sometimes it is necessary to use qualiative variable. this qualitative variable doesnot have numerical scale of measurement & so we need to defined dummy variables i.e., we assign a set of levels to qualitative variables to show its effect on responese variables, which done through the usage of dummy variable

Eg(1): x → 0 OC, 1 BC, 2 SC, 3 S/T

eg(2): suppose that a mechanical engineer wishes to relate the effective life of cutting tool (y) used on a lathe to the lathe speed in revolution per minute (x_1) & type of cutting tools used (x_2). the x_2 the tool type is qualitative has 2 levels (A or B)

⇒ We use a dummy variable that takes values of 0 or 1 to be identify the classes of regressor variable let $x_2 = 0$ if the observation is from tool type A

$x_2 = 1$ is observation from B

we can take any two distinct values parameter for x_2 in place of 0 or 1 like -1, +1 etc

However $\{0,1\}$ are usually best

* Test of Hypothesis w.r.to Regression parameter based on linear model :-

⇒ consider the simple linear model $y = \beta_0 + \beta_1 x + \varepsilon$

let $(x_i, y_i) = i = 1, 2, \ldots n$ be the sampling data

$\Rightarrow y_i = \beta_0 + \beta_1 x_i + \varepsilon$ [Normally identity distribution]

$\varepsilon \sim NID(0, \sigma^2)$

According to principles of least square $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_1 = \frac{cov(X,Y)}{V(X)} = S_{xy}/S_{xx} \Rightarrow \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$\Rightarrow (\bar{x}, \bar{y})$

$\Rightarrow (\hat{y} - \bar{y}) = \hat{\beta}_0 + \hat{\beta}_1 (x - \bar{x})$

Here $\hat{\beta}_0 = 0$ is sample linear regession passg through $(\bar{x}, \bar{y})$